6.1 Show that for a causal sequence $x[n]$ defined for $n \geq 0$ and with a z-transform $X(z)$,

$$x[0] = \lim_{z\to\infty} X(z).$$

The above result is known as the *initial value theorem.*

6.5 Consider the following sequences:
(i) $x_1[n] = (0.3)^n \mu[n+1]$, (ii) $x_2[n] = (0.7)^n \mu[n-1]$, (iii) $x_3[n] = (0.4)^n \mu[n-5]$,
(iv) $x_4[n] = (-0.4)^n \mu[-n-2]$.
(a) Determine the ROCs of the z-transform of each of the above sequences.
(b) From the ROCs determined in Part (a), determine the ROCs of the following sequences:
(i) $y_1[n] = x_1[n] + x_2[n]$, (ii) $y_2[n] = x_1[n] + x_3[n]$, (iii) $y_3[n] = x_1[n] + x_4[n]$,
(iv) $y_4[n] = x_2[n] + x_3[n]$, (v) $y_5[n] = x_2[n] + x_4[n]$, (vi) $y_6[n] = x_3[n] + x_4[n]$.

6.7 Determine the z-transform of each of the following sequences and their respective ROCs. Assume $|\beta| > |\alpha| > 0$. Show their pole-zero plots and indicate clearly the ROC in these plots.
(a) $x_1[n] = (\alpha^n + \beta^n)\mu[n+2]$, (b) $x_2[n] = \alpha^n \mu[-n-2] + \beta^n \mu[n-1]$,
(c) $x_3[n] = \alpha^n \mu[n+1] + \beta^n \mu[-n-2]$.

6.20 Each one of following z-transforms

$$X_a(z) = \frac{3z}{z^2 + 0.3z - 0.18}, \qquad X_b(z) = \frac{3z^2 + 0.1z + 0.87}{(z+0.6)(z-0.3)^2}$$

has three ROCs. Evaluate their respective inverse z-transforms corresponding to each ROC.

M 6.1 Using Program 6_1, determine the factored form of the following z-transforms:

(a) $$G_1(z) = \frac{2z^4 - 5z^3 + 13.48z^2 - 7.78z + 9}{4z^4 + 7.2z^3 + 20z^2 - 0.8z + 8},$$

(b) $$G_2(z) = \frac{5z^4 + 3.5z^3 + 21.5z^2 - 4.6z + 18}{5z^4 + 15.5z^3 + 31.7z^2 + 22.52z + 4.8},$$

and show their pole-zero plots. Determine all possible ROCs of each of the above z-transforms, and describe the type of their inverse z-transforms (left-sided, right-sided, two-sided sequences) associated with each of the ROCs.