EE E6820: Speech & Audio Processing & Recognition

Lecture 2: Acoustics

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1. The wave equation
2. Acoustic tubes: reflections & resonance
3. Oscillations & musical acoustics
4. Spherical waves & room acoustics

Outline

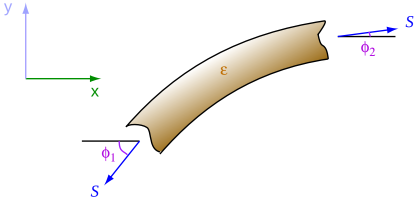
1. The wave equation
2. Acoustic tubes: reflections & resonance
3. Oscillations & musical acoustics
4. Spherical waves & room acoustics

Acoustics & sound

- Acoustics is the study of physical waves
- (Acoustic) waves transmit energy without permanently displacing matter (*e.g.* ocean waves)
- Same math recurs in many domains
- Intuition: pulse going down a rope

The wave equation

Consider a small section of the rope



Displacement $y(x)$, tension S, mass $\epsilon\, dx$
$\Rightarrow$ Lateral force is

$$F_y = S\sin(\phi_2) - S\sin(\phi_1)$$
$$\approx S\frac{\partial^2 y}{\partial x^2}\,dx$$

Wave equation (2)

Newton's law: $F = ma$

$$S\frac{\partial^2 y}{\partial x^2}\,dx = \epsilon\,dx\frac{\partial^2 y}{\partial t^2}$$

Call $c^2 = S/\epsilon$ (tension to mass-per-length)
hence, the Wave Equation:

$$c^2\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

. . . partial DE relating curvature and acceleration

Solution to the wave equation

If $y(x, t) = f(x - ct)$ (any $f(\cdot)$)
then

$$\frac{\partial y}{\partial x} = f'(x - ct) \qquad \frac{\partial y}{\partial t} = -cf'(x - ct)$$

$$\frac{\partial^2 y}{\partial x^2} = f''(x - ct) \qquad \frac{\partial^2 y}{\partial t^2} = c^2 f''(x - ct)$$

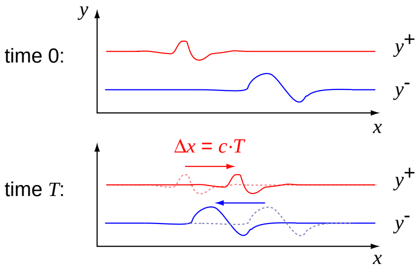
also works for $y(x, t) = f(x + ct)$
Hence, general solution:

$$c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow y(x, t) = y^+(x - ct) + y^-(x + ct)$$

Solution to the wave equation (2)

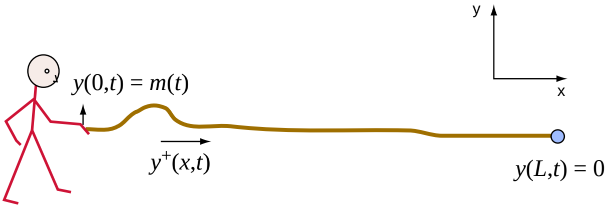
- $y^+(x - ct)$ and $y^-(x + ct)$ are traveling waves
  - shape stays constant but changes position



- c is traveling wave velocity $(\Delta x / \Delta t)$
- y^+ moves right, y^- moves left
- resultant $y(x)$ is sum of the two waves

Wave equation solutions (3)

- What is the form of y^+, y^-?
  - any doubly-differentiable function will satisfy wave equation
- Actual waveshapes dictated by boundary conditions
  - *e.g.* $y(x)$ at $t = 0$
  - plus constraints on y at particular xs
    e.g. input motion $y(0, t) = m(t)$
    rigid termination $y(L, t) = 0$

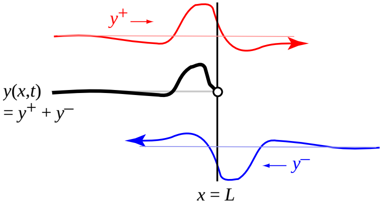


Terminations and reflections

- System constraints:
  - initial $y(x, 0) = 0$ (flat rope)
  - input $y(0, t) = m(t)$ (at agent's hand) ($\rightarrow y^+$)
  - termination $y(L, t) = 0$ (fixed end)
  - wave equation $y(x, t) = y^+(x - ct) + y^-(x + ct)$
- At termination:
  - $y(L, t) = 0 \Rightarrow y^+(L - ct) = -y^-(L + ct)$

i.e. y^+ and y^- are mirrored in time and amplitude around $x = L$

$\Rightarrow$ inverted reflection at termination

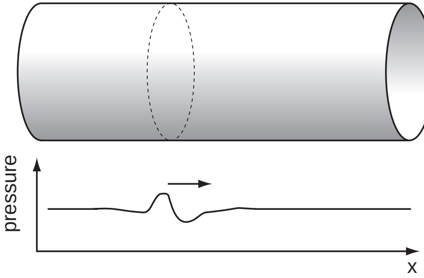


[simulation `travel1.m`]

Outline

Acoustic tubes

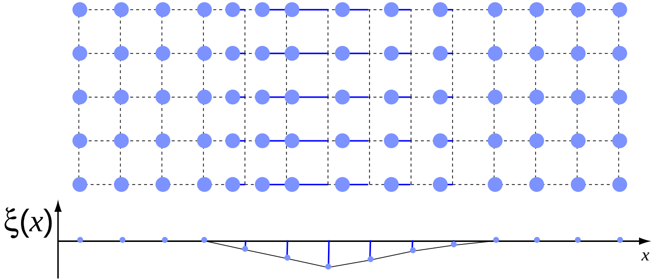
- Sound waves travel down acoustic tubes:



  - 1-dimensional; very similar to strings
- Common situation:
  - wind instrument bores
  - ear canal
  - vocal tract

Pressure and velocity

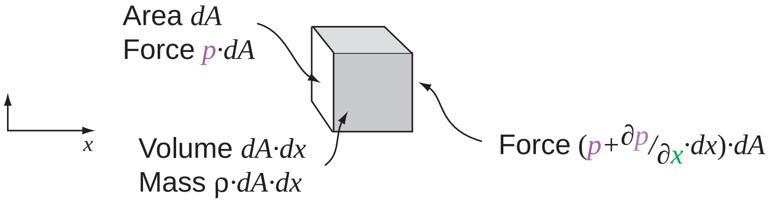
- Consider air particle displacement $\xi(x, t)$



- Particle velocity $v(x, t) = \frac{\partial \xi}{\partial t}$
- hence volume velocity $u(x, t) = Av(x, t)$
- (Relative) air pressure $p(x, t) = -\frac{1}{\kappa}\frac{\partial \xi}{\partial x}$

Wave equation for a tube

- Consider elemental volume



- Newton's law: $F = ma$

$$-\frac{\partial p}{\partial x}\, dx\, dA = \rho\, dA\, dx \frac{\partial v}{\partial t}$$

$$\Rightarrow \frac{\partial p}{\partial x} = -\rho \frac{\partial v}{\partial t}$$

$$\therefore c^2 \frac{\partial^2 \xi}{\partial x^2} = \frac{\partial^2 \xi}{\partial t^2} \qquad c = \frac{1}{\sqrt{\rho \kappa}}$$

Acoustic tube traveling waves

- Traveling waves in particle displacement:

$$\xi(x,t) = \xi^+(x-ct) + \xi^-(x+ct)$$

- Call $u^+(\alpha) = -cA\frac{\partial}{\partial\alpha}\xi^+(\alpha)$, $Z_0 = \frac{\rho c}{A}$
- Then volume velocity:

$$u(x,t) = A\frac{\partial \xi}{\partial t} = u^+(x-ct) - u^-(x+ct)$$

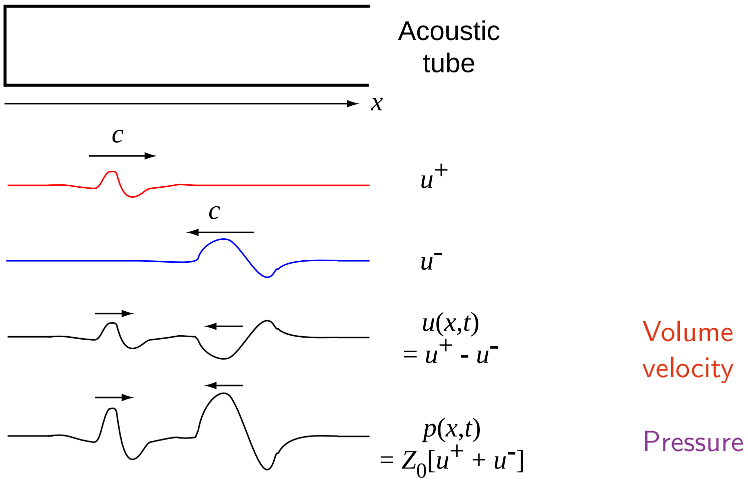
- And pressure:

$$p(x,t) = -\frac{1}{\kappa}\frac{\partial \xi}{\partial x} = Z_0\left[u^+(x-ct) + u^-(x+ct)\right]$$

- (Scaled) sum and difference of traveling waves

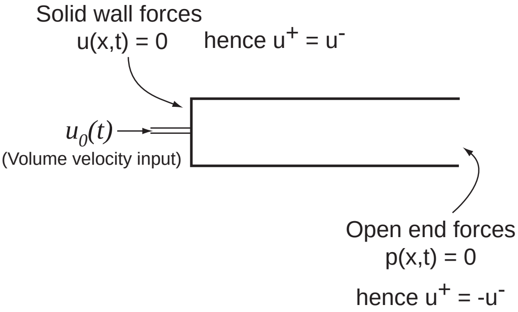
Acoustic traveling waves (2)

Different resultants for pressure and volume velocity:



Terminations in tubes

- Equivalent of fixed point for tubes?



- Open end is like fixed point for rope: reflects wave back inverted
- Unlike fixed point, solid wall reflects traveling wave without inversion

Standing waves

- Consider (complex) sinusoidal input

$$u_0(t) = U_0 e^{j\omega t}$$

- Pressure/volume must have form $Ke^{j(\omega t+\phi)}$
- Hence traveling waves:

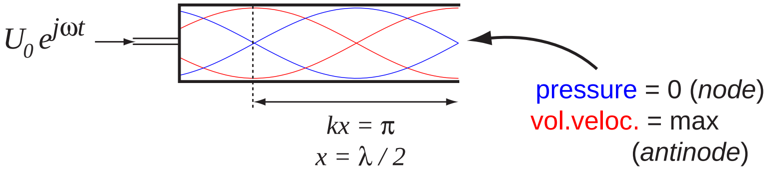
$$u^+(x - ct) = |A| e^{j(-kx+\omega t+\phi_A)}$$
$$u^-(x + ct) = |B| e^{j(kx+\omega t+\phi_B)}$$

  where $k = \omega/c$ (spatial frequency, rad/m)
  (wavelength $\lambda = c/f = 2\pi c/\omega$)

- Pressure and volume velocity resultants show stationary pattern: standing waves
  - even when $|A| \neq |B|$

  $\Rightarrow$ [simulation `sintwavemov.m`]

Standing waves (2)



- For lossless termination ($|u^+| = |u^-|$), have true nodes and antinodes
- Pressure and volume velocity are phase shifted
  - in space and in time

Transfer function

Consider tube excited by $u_0(t) = U_0 e^{j\omega t}$

- sinusoidal traveling waves must satisfy termination 'boundary conditions'
- satisfied by complex constants A and B in

$$\begin{aligned} u(x,t) &= u^+(x-ct) + u^-(x+ct) \\ &= Ae^{j(-kx+\omega t)} + Be^{j(kx+\omega t)} \\ &= e^{j\omega t}(Ae^{-jkx} + Be^{jkx}) \end{aligned}$$

- standing wave pattern will scale with input magnitude
- point of excitation makes a big difference ...

Transfer function (2)

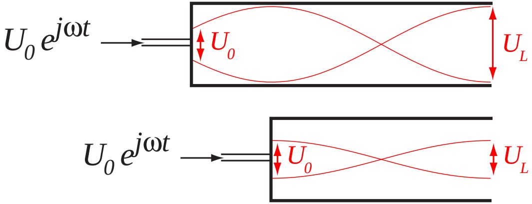
For open-ended tube of length L excited at $x = 0$ by $U_0 e^{j\omega t}$

$$u(x,t) = U_0 e^{j\omega t} \frac{\cos k(L-x)}{\cos kL} \qquad k = \frac{\omega}{c}$$

- (matches at $x = 0$, maximum at $x = L$)

i.e. standing wave pattern

- *e.g.* varying L for a given ω (and hence k):

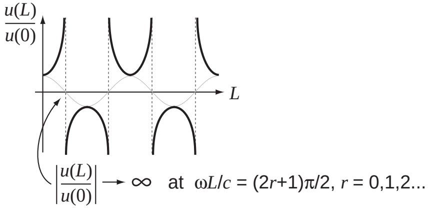


- magnitude of U_L depends on L (and ω)

Transfer function (3)

- Varying ω for a given L, *i.e.* at $x = L$

$$\frac{U_L}{U_0} = \frac{u(L,t)}{u(0,t)} = \frac{1}{\cos kL} = \frac{1}{\cos(\omega L/c)}$$

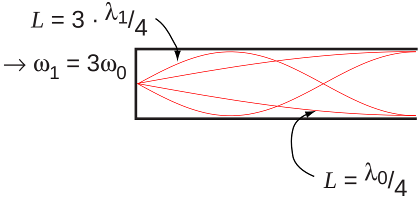


- Output volume velocity always larger than input
- Unbounded for $L = (2r+1)\frac{\pi c}{2\omega} = (2r+1)\frac{\lambda}{4}$
  i.e. resonance (amplitude grows without bound)

Resonant modes

For lossless tube with $L = m\frac{\lambda}{4}$, m odd, λ wavelength
$\left|\frac{u(L)}{u(0)}\right|$ is unbounded, meaning:

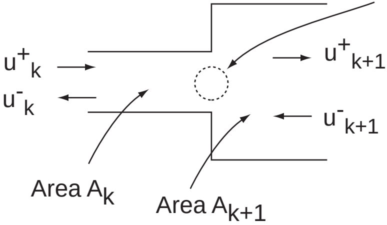
- transfer function has pole on frequency axis
- energy at that frequency sustains indefinitely



- compare to time domain ...

e.g. 17.5 cm vocal tract, $c = 350$ m/s
$\Rightarrow \omega_0 = 2\pi\,500$ Hz (then 1500, 2500, ...)

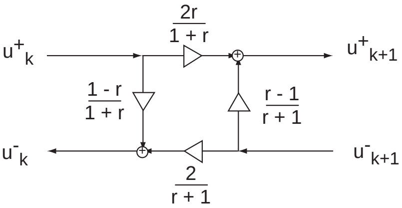
Scattering junctions



At abrupt change in area:

- pressure must be continuous
  $p_k(x, t) = p_{k+1}(x, t)$
- vol. veloc. must be continuous
  $u_k(x, t) = u_{k+1}(x, t)$
- traveling waves
  $u^+_k, u^-_k, u^+_{k+1}, u^-_{k+1}$
  will be different

Solve *e.g.* for u_k^- and u_{k+1}^+: (generalized term)

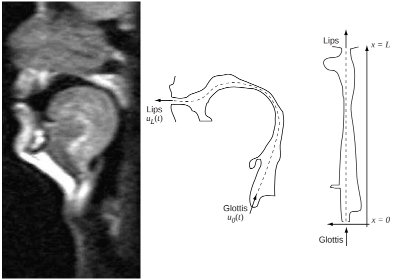


$$r = \frac{A_{k+1}}{A_k}$$

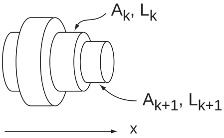
"Area ratio"

Concatenated tube model

Vocal tract acts as a waveguide

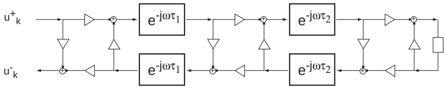


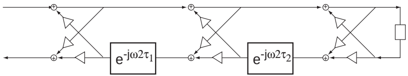
Discrete approximation as varying-diameter tube



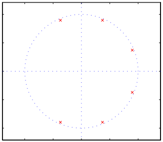
Concatenated tube resonances

Concatenated tubes → scattering junctions → lattice filter





Can solve for transfer function – all-pole

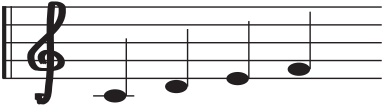


Approximate vowel synthesis from resonances
[sound example: ah ee oo]

Outline

Oscillations & musical acoustics

Pitch (periodicity) is essence of music



- why? why music?

Different kinds of oscillators

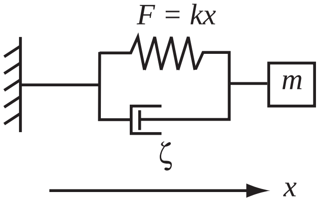
- simple harmonic motion (tuning fork)
- relaxation oscillator (voice)
- string traveling wave (plucked/struck/bowed)
- air column (nonlinear energy element)

Simple harmonic motion

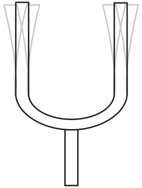
- Basic mechanical oscillation

$$\ddot{x} = -\omega^2 x \qquad x = A\cos(\omega t + \phi)$$

- Spring + mass (+ damper)



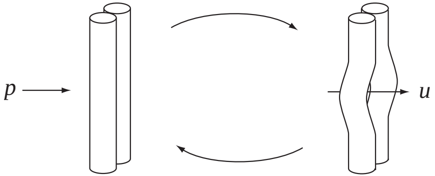
$$\omega^2 = \frac{k}{m}$$



- *e.g.* tuning fork
- Not great for music
  - fundamental ($\cos \omega t$) only
  - relatively low energy

Relaxation oscillator

- Multi-state process
  - one state builds up potential (*e.g.* pressure)
  - switch to second (release) state
  - revert to first state, etc.
- *e.g.* vocal folds:

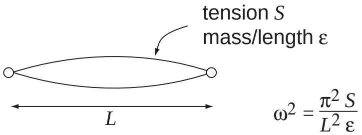


http://www.youtube.com/watch?v=ajbcJiYhFKY

- Oscillation period depends on force (tension)
  - easy to change
  - hard to keep stable
  - ⇒ less used in music

Ringing string

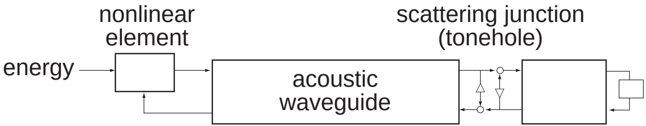
- *e.g.* our original 'rope' example



- Many musical instruments
  - guitar (plucked)
  - piano (struck)
  - violin (bowed)
- Control period (pitch):
  - change length (fretting)
  - change tension (tuning piano)
  - change mass (piano strings)
- Influence of excitation ... [pluck1a.m]

Wind tube

- Resonant tube + energy input



$$\omega = \frac{\pi c}{2L}$$ (quarter wavelength)

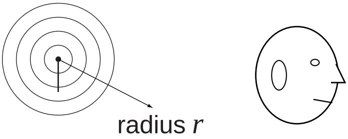
- *e.g.* clarinet
  - lip pressure keeps reed closed
  - reflected pressure wave opens reed
  - reinforced pressure wave passes through
- finger holds determine first reflection

  $\Rightarrow$ effective waveguide length

Outline

Room acoustics

- Sound in free air expands spherically:



- Spherical wave equation:

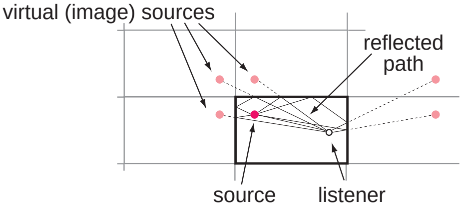
$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

  solved by $p(r,t) = \frac{P_0}{r} e^{j(\omega t - kr)}$

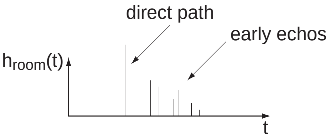
- Energy $\propto p^2$ falls as $\frac{1}{r^2}$

Effect of rooms (1): Images

Ideal reflections are like multiple sources:



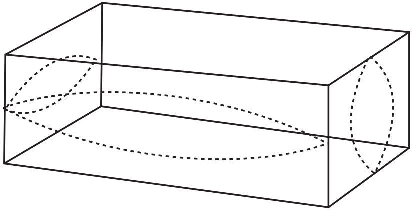
'Early echoes' in room impulse response:



- actual reflections may be $h_r(t)$, not $\delta(t)$

Effect of rooms (2): Modes

Regularly-spaced echoes behave like acoustic tubes

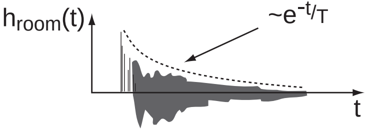


Real rooms have lots of modes!

- dense, sustained echoes in impulse response
- complex pattern of peaks in frequency response

Reverberation

- Exponential decay of reflections:



- Frequency-dependent
  - greater absorption at high frequencies
  - ⇒ faster decay
- Size-dependent
  - larger rooms → longer delays → slower decay
- Sabine's equation:

$$RT_{60} = \frac{0.049V}{S\bar{\alpha}}$$

- Time constant varies with size, absorption

Summary

- Traveling waves
- Acoustic tubes & resonances
- Musical acoustics & periodicity
- Room acoustics & reverberation

Parting thought

- Musical bottles

References